REVIEWS AND ABSTRACTS OF LITERATURE

Die logischen Grundlagen der exakten Wissenschaften. PAUL NATORP. Leipzig: B. G. Teubner. 1910. Pp. xx + 416.

In the relation between philosophy and science periods of intimate *rapprochements* seem to alternate with periods of mutual distrust. Professor Natorp suggests that the true interrelations between the two are lost sight of in periods in which each is developing along established lines, but come to light in those critical periods in which either is compelled to strike out into new lines. At any rate, it is well to note that the traditional distrust of metaphysics on the part of scientists has recently been rapidly disappearing. The radical reconstruction of physical theory necessitated by recent research has driven physicists into philosophy almost in spite of themselves. As a token of this trend of the times, the firm of Teubner is now publishing a series of books under the heading "Science and Hypothesis," the contributors to which are such distinguished scientists as Poincaré, Planck, Enriques, and Picard.

It is a pleasure to welcome, in behalf of American readers, this excellent series, well printed, moderately priced, and gotten up in the handy form for which Teubner is justly famous.

The volume before us is the twelfth in the series, and represents the effort of a certain school of philosophy to reinterpret the meaning of mathematics and mechanics. The orthodox neo-Kantian movement, of which Herman Cohen is the Nestor, has always interpreted the Kantian metaphysics as the logical foundation of Newton's "Principia"; and of late the Marburg School, of which Cohen and Natorp are the leaders, and E. Cassirer in some respects a most brilliant ally, has been especially active in tracing in detail the logical structure of the mathematical and physical sciences. Professor Natorp has for the past twelve years been publishing a series of papers and booklets on the philosophy of mathematics, and the work before us is the definitive form of this neo-Kantian philosophy of the exact sciences. In consonance with its aim to be a modern "Critique of Pure Reason," the book seeks to be constructive throughout and *streng wissenschaftlich* in the classic sense. Still, the author does not disdain, in passing, to show us the frailties of the neo-Leibnitzians who have recently been hammering at the Kantian philosophy of mathematics in the most merciless manner. We may, therefore, also regard the book as the neo-Kantian counterblast to Russell's "Principles of Mathematics" or to Couturat's version of it.

Chapter I. is devoted to the consideration of "The Problem of a Logic of the Exact Sciences." As Professor Natorp views it, the situation resembles very much the one that confronted Kant. On one side are the empiricists, represented by Kronecker and Helmholtz, and on the other hand the neo-Leibnitzian school of Frege, Russell, etc., who would reduce mathematics to pure logic. Professor Natorp, like Kant, rejects both of these positions, holding fast both to the *a priori* and the synthetic char-

acter of mathematical knowledge. He does not, however, follow Kant in attributing the peculiar certainty of mathematics to an *a priori* intuition. Instead he bases it on the synthetic process of thought. This, of course, will appear to many as an abandonment of the distinctively Kantian attitude, and as opening the floodgates of Hegelian panlogism which was so foreign to Kant. Professor Natorp, however, while a professed and thoroughgoing idealist, is tenacious in his adherence to the transcendental method and to the distinction between analytic and synthetic thought.

Dismissing the empiricists as not even worthy of the courtesy of a refutation, our author introduces his own point of view through a criticism of the analytic or "formalist" school. The error of this view is traced to the "dogmatic" Aristotelian logic which proposes to define everything until we come to the indefinable and prove everything until we get to the undemonstrable. This in turn is based on the fundamental error of naïve realism which regards things as given in perception, and conceives the work of thought to consist in the analytic working over of the content of perception. In opposition Professor Natorp holds to the Kantian dictum: No analysis without previous synthesis. As we can not observe this synthesis in action, it is attributed to a "primitive understanding" (p. 9). About the metaphysic of this primitive understanding, where, when, and what it is, *e. g.,* whether it is an individual or universal mind, etc., no revelations are made to us. The only thing the author is willing to tell us is that it is not in time, and is beyond the ken of the psychologist (*cf.* p. 99). The uncertainty in which we are left as to the whereabouts of this primitive synthesis, prevents us from understanding Professor Natorp's attitude to ordinary logic. On one hand he tells us that ordinary logic is analytic and can not extend our knowledge. Like the microscope, it can widen the angle between the rays, but can not increase their number. On the other hand, he admits that Frege, Couturat, and even Wolff and the older Leibnitzians mean by analysis something which does extend our knowledge. Indeed he is forced to use his own example of the microscope and admit that that instrument can materially add to our knowledge.

The effort to find a starting-point for his logic gives our author—not to mention the reader—considerable trouble. The transcendental method consists in following presuppositions. But there are no presuppositions unless something is given. What *is* given? It can not be the object of perception, for that is precisely the thing which it is the aim of knowledge to determine. After a long discussion, in which a great deal of emphasis is laid on infinite process, *genetisch* as opposed to *ontisch,* the *fieri* as opposed to the *factum,* we reach the conclusion that the problem of logic (*i. e.,* the transcendental variety) is to find those presuppositions which are necessary to bring the x—the undetermined, but to-be-determined, object of experience—into complete determination. This complete determination, we are assured almost *ad infinitum,* can be reached only by an infinite process.

In the second chapter we have a modernized deduction of the categories.

The dry bones of the Kantian framework receive a great deal of flesh and blood. In the end, however, they turn out to be our old friends the Twelve, marching in four groups of three each. If it were not for the fact that students at our colleges *do not* read German, this chapter could profitably be recommended to those who are reading Kant for the first time and who generally can not grasp what these categories are about.

In the third chapter we have a deduction of the number concept and of the four rules of arithmetic. The first condition, we are told, for the understanding of number is not to have anything to do with given things, for the latter already presuppose number. We must deal with rules of thought. Thought consists in nothing but positing relations, and the terms between which the relations hold are subsequent to the positing activity. Thus there is built up a fundamental series from which the number series is deduced. The issue of priority between ordinal and cardinal numbers is settled by calling them correlative. The mooted question as to whether the idea of number is dependent on time and space is answered mainly in the negative. Professor Natorp, however, thinks he saves something of the Kantian position by insisting that the relation of *before and after* is the common basis of number as well as of time and space. In his analysis of the operations of arithmetic he follows Simon, without, however, fully subscribing to the latter's stark subjectivism. Natorp admits that it is not enough to call numbers mental objects. We must show how they help us to cognize objects.

The critical work in this chapter is, whatever one may think of the constructive part, decidedly unsatisfactory. There is no attempt to come to close quarters with Russell's or Whitehead's definitions of numbers or their operations. Frege is taken as typical of the whole school, and arguments are used against him which Russell specifically answers. Many readers, however, who can not grow enthusiastic about the application of transcendental logic in this field, will agree with Professor Natorp in his insistence that not only are the so-called real numbers (*i. e.,* surds) and fractions to be looked on as operations, but even the series of positive integers must be so considered.

Chapter IV. is devoted to "Continuity and Infinity" and Chapter V. to "Direction and Dimension" as terminations of pure number. The modern account of infinity and continuity is accepted, but Professor Natorp remains loyal to Herman Cohen and insists on the notion of the infinitesimal. By means of this he builds up the idea of the *reality of something* which forms a transition from mathematics to mechanics. The specific criticisms of Russell against Cohen's use of the infinitesimal method are not directly answered. Indeed, so far as the Marburg school is concerned, the great work of Weierstrass might as well never have been accomplished.

Chapter VI. is entitled: "Time and Space as Mathematical Structures (*Gebilde*)." In the discussion of time and space the author adheres, in the main, to the Kantian view. He would, however, change the Kantian order somewhat, and make time and space refer back to the categories of modality, relation, etc. The main point seems to be the insistence that

time and space are more than number in so far as they give existential reference to that which otherwise would be purely mathematical.

In the discussion of geometry, Professor Natorp no longer contends, as he did a few years ago, that metric geometry can not be subordinated to projective geometry; and he also seems to weaken somewhat in his hopeless stand that non-Euclidean geometry contradicts the fundamental axioms of the continuity and homogeneity of space. Logically, he reluctantly admits, non-Euclidean geometry is possible. He rejects it, however, on the philosophical ground that a space of more than three dimensions or of non-Euclidean constitution would lead to endless indeterminateness, and make existential reference impossible. Why a three-dimensional Riemannian space, or even a four-dimensional mechanics as recently sketched by Minkowski, should be considered any more indeterminate than Euclidean space or Newtonian mechanics, the present reviewer can not understand. A friend, however, makes the perhaps irrelevant suggestion that no man over forty-five will ever admit the possibility of a system of geometry other than the one which he was taught when a boy.

The last chapter—perhaps the most interesting—is entitled: "The Temporo-spatial Order of Phenomena and the Mathematical Principles of Natural Science." In the discussion of the question of absolute time and space, Mach's arguments against Newton are easily turned around to fortify the Kantian position. Absolute time and space are not found *in* experience precisely because they are the very conditions which make experience possible. Mach's argument that absolute time and space are not real things and, therefore, of no practical importance, is met by the observation that in the same way nothing in our experience is absolutely one, yet the laws of arithmetic based on abstract numbers are assuredly of some practical importance. Natorp's criticism of Mach would have been more effective if the former were in a position to analyze the latter's conception of existence (in the assertion that only relative motion *exists*); but Natorp's own conception of existence is, like that of most philosophers, entirely vague. For the most part he holds existence to be equivalent to complete determination (*cf.* pp. 336, 338). As we are repeatedly told that the process of determination is infinite, it would seem that the *existence* of things is the one thing forever unattainable to us.

The remainder of the last chapter is devoted to an epistemologic deduction of the fundamental laws of mechanics. Newton's laws of motion, and even the principle of the conservation of energy, are all shown to be necessary on the principles of transcendental logic. It is not likely, however, that Messrs. Abraham, Lewis, Bucherer, and the others will take those proofs so seriously as to discontinue their labors in the direction of a non-Newtonian mechanics which should meet the facts of physics more adequately than does the classic mechanics.

In the last two paragraphs of the book we have one of the first attempts to determine the philosophic value of the relativity theory of Einstein and Minkowski. There is, however, no attempt to discuss the point wherein this theory does most violence to traditional views, viz., its conception of what constitutes simultaneity.

The book before us is German in more senses than one. It is thorough and packed with information and close reasoning. The author has spent considerable labor on the great mathematicians—though one suspects that the intercourse has been too Platonic, *i. e.*, chaste and unfruitful. Professor Natorp does not seem to know the English or Italian works on symbolic logic. He apparently has not read Russell's book on Leibnitz; indeed, he has not read Russell's "Principles of Mathematics" with great care, if we are to judge by his references to Russell's views on analytic and synthetic judgments. There is also no reference to such French works on the theory of science as those of Picard or Duhem. There are, indeed, a few references to the German translation of Poincaré's "Science and Hypothesis," but the fundamental thesis of that work is not dealt with. Indeed, from the point of view which looks upon the fundamental principles of mathematics as hypotheses justified only by the fact that they give us a coherent scientific system, a good deal of Natorp's work as to the foundation of these principles must appear as entirely uncalled for.

Two closely related methods are typified in this book, which are characteristic of classic Hegelian philosophy and which have caused the latter to fall into such sad repute. These are (1) the method of dealing with the implication of *concepts* (not to be confused with the implication of propositions), and (2) the covert appeal to the self-evident. These methods are due to the prevailing belief that the relation between any two ideas is a relatively simple affair, which does not, like the relation between natural objects, need elaborate investigation. It is one of the great services rendered by mathematics to have shown that the relation between ideas requires long and patient inquiry, in which we are to be especially on our guard against any appeal to the apparently obvious. In a question like the convergency of a series, that which most people would regard as obvious turns out after laborious investigation to be almost invariably wrong.

We must, however, agree with Professor Natorp that there is much less danger to-day of Hegelian intrusions into the special sciences than there is of ignoring the fundamental problems of philosophy and of substituting for them an easy mixture of propositions from biology or physics seasoned with logically loose and vague general reflections. Such seem to me Ostwald's "Naturphilosophie," and, in large measure, a good deal of the work of Avenarius and Bergson.

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VIme congrès international de psychologie: rapports et comptes rendus. ED. CLAPARÈDE (Editor). Geneva: Kündig. 1910. Pp. 877.

It is impossible to attempt, in a short review, a summary of the substance of the various papers and discussions given at this congress, held in 1909, and reported in the foregoing large-paged, bulky volume. It may be of interest, however, to indicate the matters that the directing committee deemed at that time chiefly important for discussion, by listing the topics, with the number of pages in the report devoted to each. (1) "The